

Viktor Dremin*¹, Evgeny Zherebtsov^{1,2}, Andrey Dunaev¹,
Alexey Popov³, Alexander Bykov³ and Igor Meglinski³

¹ Scientific-Educational Center “Biomedical Engineering”,
Orel State University named after I.S. Turgenev, Orel, Russia

² Optoelectronics and Biomedical Photonics Group, Aston University, Birmingham, UK

³ Optoelectronics and Measurement Techniques Laboratory, University of Oulu, Oulu, Finland

*Contact: dremin_viktor@mail.ru

Keywords: fluorescence, modeling, diffusion theory

This paper is devoted to the development of a mathematical model of the propagation of fluorescent radiation through a human skin. Based on the results of the review and analysis, the most promising of the models considered is the model based on the solution of the diffusion equation [1].

The biotissue is taken as a two-layer medium consisting of a layer of epidermis of thickness D and a semi-infinite dermis. The total fluorescence of the tissue can be calculated as the sum of the contributions from the two layers. Within each layer, the model should take into account the propagation of the excitation light I_0 , absorption coefficient μ_{af} , quantum efficiency ϕ_f of fluorophores and optical parameters of the layers.

Two different models were considered to describe the propagation of light in each layer. Taking into account that the mean free path of a photon for the epidermis is comparable with its thickness (~ 100 - 200 μm), the propagation of light can be approximated by means of the Beer law. In contrast, the dermis layer has a relatively high level of scattering and a small value of the mean free path of the photon. Under these conditions, diffusion theory can provide a good description of light propagation.

The total fluorescence F_{skin} is the sum of four main components. F_1 is fluorescence directed toward the surface of the tissue directly from the epidermis itself. Part of the fluorescence of the epidermis reaches the dermis F_{ep}^+ , diffusely reflects and returns to the surface of the tissue F_2 . It can be specified using the ID diffusion theory with the δ -Eddington approximation [2, 3]. Diffusively reflected excitation light from the dermis I_0^- excites additional fluorescence of the epidermis F_3 . Excitation light I_0^+ also initiates fluorescence in the dermis. Using the ID diffusion theory, dermal fluorescence, attenuated on the epidermal-dermal border and passing through the epidermis, can be represented in the form of F_4 [4].

$$\left\{ \begin{array}{l} F_1 = \frac{1}{2} R_{NA} \int_0^D I_0 \cdot \exp[-\mu_{d1}(\lambda_{ex}) \cdot z] \times \sum_{k=1}^N [\mu_{af1-k} \cdot \phi_{f1-k}] \cdot \exp(-\mu_{d1}(\lambda_{em}) \cdot z) dz, \\ F_2 = R_{NA} \cdot \pi \cdot [A(\lambda_{em}) + C_2(\lambda_{em})] - \frac{3\mu_{s1}(\lambda_{em}) \cdot g_2 \cdot F_{ep}^+ + 4\pi \cdot [A(\lambda_{em}) \cdot \mu_{t2}(\lambda_{em}) + C_2(\lambda_{em}) \cdot \mu_{d2}(\lambda_{em})]}{6(\mu_{a2}(\lambda_{em}) + \mu'_{s2}(\lambda_{em}))} \cdot \exp[-\mu_{d1}(\lambda_{em}) \cdot D], \\ F_3 = R_{NA} \int_0^D I_0^- \cdot \exp[-\mu_{d1}(\lambda_{ex}) \cdot z] \times \sum_{k=1}^N [\mu_{af1-k} \cdot \phi_{f1-k}] \cdot \exp[-\mu_{d1}(\lambda_{em}) \cdot (D-z)] dz, \\ F_4 = [R_{NA} \cdot \mu_{af2} \cdot \phi_{f2} \cdot \left[\frac{4\pi \cdot A(\lambda_{ex}) + I_0^+}{\mu_{t2}(\lambda_{ex}) + \mu_{d2}(\lambda_{em})} + \frac{4\pi \cdot C_2(\lambda_{ex})}{\mu_{d2}(\lambda_{ex}) + \mu_{d2}(\lambda_{em})} \right] \cdot \frac{3(\mu_{a2}(\lambda_{em}) + \mu'_{s2}(\lambda_{em}))}{3(\mu_{a2}(\lambda_{em}) + \mu'_{s2}(\lambda_{em})) + 2\mu_{d2}(\lambda_{em})}] \cdot \exp[-\mu_{d1}(\lambda_{em}) \cdot D]. \end{array} \right.$$

Fraction of light collected by the fiber-optic probe is approximated as the ratio R_{NA} of the solid angle subtended by the fiber numerical aperture to hemispherical collection solid angle $\pi/2$. A and C_2 are the general and particular solutions of the diffusion equation [2].

Such a two-layer model with the use of fitting algorithms for experimental data can allow solving the inverse problem of estimating the concentration of fluorophores.

1. S.K. Chang, D. Arifler, R. Drezek, M. Follen, R. Richards-Kortum, “Analytical model to describe fluorescence spectra of normal and preneoplastic epithelial tissue: comparison with Monte Carlo simulations and clinical measurements,” *Journal of Biomedical Optics* **3**, pp. 511–522, 2004
2. A. Ishimaru, “Wave Propagation and Scattering in Random Media” (Academic Press, 1978)
3. W.M. Star, “Diffusion theory of light transport” (Plenum Press, 1995)
4. J. Wu, M.S. Feld, R.P. Rava, “Analytical model for extracting intrinsic fluorescence in turbid media,” *Applied Optics* **19**, pp. 3585–3595, 1993